Government 2005
Formal Political Theory I
Lecture 1

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This course

- Topics:
  - Introduction to game-theoretic toolkit
  - Applications in political science & political economy
- Goals:
  - Students lose fear of game theory
  - Students acquire working knowledge of games of complete (and easy incomplete) information
  - First in a two-course sequence
- Evaluation:
  - Problem sets (40%)
  - Final exam (40%)
  - Paper (20%)

Today's class

- What is game theory?
  - Why is it used in political science?
  - The rational choice controversy
- What is a game?
  - Basic definition
  - Normal (or strategic) form vs. extensive form
  - Classification(s) of games
- Let's play ball! Simple (but useful) games
- Pure-strategy Nash equilibrium

Game theory

- Game theory $\Rightarrow$ formal analysis of the behavior of interacting decision makers
  - Decision theory = branch of math analyzing decision problem of single individual (external environment as primitive)
  - Game theory = interactive decision theory
- **Strategic interdependence** $\Rightarrow$ each individual's welfare depends on her actions + others' actions. And therefore her best actions depend on what she expects the others to do
- A few uses of game theory:
  - How much money lobbies donate to influence policy making
  - How politicians choose platforms to win elections
  - How legislators bargain over policy
  - Allocation of troops and arms in battles and wars
  - How we can signal our ability to prospective employers
  - Whether protesters should join street demonstrations

Rational choice

- Rational choice $\Rightarrow$ part of many models in game theory
  - Decision maker chooses best action based on her preferences
  - No qualitative restriction on preferences
  - Enough to assume $\Rightarrow$ **completeness** + **consistency**
  - Complete prefs: $a \succ_i b$ or $a \prec_i b$ or $a \sim_i b$, $\forall i$ and $\forall (a, b) \in A_i$
  - Consistent (or transitive) prefs: if $a \succ b$ and $b \succ c \Rightarrow a \succ c$

- No cycles and no effect of irrelevant alternatives, but it can accommodate: altruism, envy, myopic behavior

- Utility function as "preference indicator function"
  - $u(a) > u(b)$ iff i prefers a to b ($a \succ_i b$)
  - Only ordinal information (no intensity)

- How meaningful? It depends on the purpose. No theory right or wrong, some useful
  - E.g., London's subway map
  - E.g., Newtonian (vs relativistic) mechanics

Methodological individualism

- Claim: game theory should be used in formal models of social sciences that adhere to methodological individualism
  - Explain social phenomena as the results of the actions of many agents (chosen according to some consistent criterion)
- Max Weber (*Economy and Society*, 1922), taking about social collectivities, such as states, associations, social groups:
  - "*In sociological work collectivities must be treated as solely the resultants and modes of organization of the particular acts of individual persons, since these alone can be treated as agents in a course of subjectively understandable action*"
- Without explaining why people do what they do, hard to understand larger social phenomena
- This doesn't mean to privilege individual over collective, but privilege the **action-theoretic** level of explanation

What is a game?

A game has these elements:

(1) Set of players I $(i = 1, ..., I)$
(2) Set of actions A_i
(3) Set of outcomes Y
(4) Extensive form ϵ, determining set of possible paths of play Z
(5) Outcome function $g : Z \to Y$
(6) Preferences over outcomes $v_i : Y \to \mathbb{R}$

(1)-(5) are the **rules of the game**
(6) usually utility functions

Extensive vs normal form representation

- Extensive-form representation uses **game tree** to specify rules of the game (1)-(5) by means of *decision nodes* and *branches*, and includes payoffs (6) of all players in *terminal nodes*
- Crucial element: **information set**, defined as collection of decision nodes at which the player doesn't know where she exactly is when she moves
- Normal-form (or strategic-form) representation rests on the concept of **strategy** $\rightarrow$ Complete contingent plan that says what a player will do at each of her information sets. Formally:

Define $\mathcal{H}_i$ as set of i's information sets, $\mathcal{A}$ set of possible actions, $C(H) \subset \mathcal{A}$ subset of actions possible at information set H. Strategy for i is a function $s_i : \mathcal{H}_i \rightarrow \mathcal{A}$ such that $s_i(H) \in C(H)$, $\forall H \in \mathcal{H}_i$

Extensive vs strategic form representation (contd.)

- In I-player game, convenient to represent a profile of players' strategy choices by means of single vector: $s = (s_1, ..., s_I)$, or in short $s = (s_i, s_{-i})$
- Pure-strategy profile s belongs to the strategy space S: $s \in S = S_1 \times ... \times S_I$ (and $s_i \in S_i$)
- Normal form representation describes a game in terms of strategies and their associated payoffs. Formally:

For a game with I players, the normal form representation Γ_N specifies a set of strategies S_i for each i and a payoff function $u_i(s_1, ..., s_I)$ giving the utility levels associated with the (possibly random) outcome arising from $(s_1, ..., s_I)$. That is: $\Gamma_N = \langle I, S_i, u_i(.) \rangle$

- This definition rests on definition of **pure** strategies, we'll easily extend it as soon as we define **mixed** strategies

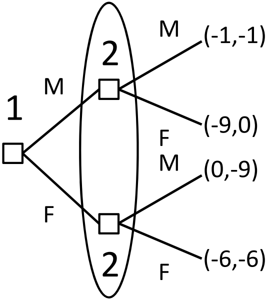
Example: Prisoner's dilemma

		Prisoner 2	
		Mum	Fink
Prisoner 1	Mum	(-1,-1)	(-9, 0)
	Fink	(0,-9)	(-6,-6)

- Normal form representation of the game is a specification of players, players' strategy spaces, and players' payoff functions
  - Players: Prisoner 1 and prisoner 2
  - Strategy spaces: Mum, Fink
  - Payoff functions: As indicated by payoff matrix

Example: Prisoner's dilemma (contd.)

- Extensive-form representation of the game uses game tree:



- The circle captures the information set of prisoner 2 (initial node is the information set of prisoner 1). If all information sets are singleton, we have game of perfect information

Classification(s) of games

1. Cooperative vs non-cooperative games
   - **Cooperative** game theory does not model bargaining, but considers how much surplus each coalition of players can get with binding agreement, and division of surplus that may arise
   - **Non-cooperative** game theory assumes binding agreement are not feasible, or that the bargaining process leading to a binding agreement is formalized in a larger game
   - Non-cooperative game theory is not the study of non-cooperative behavior, but rather a method of analysis

2. Static vs dynamic games
   - **Static** = each player moves once and all players move *simultaneously* (or with no information on others' moves)
   - **Dynamic** = moves are *sequential* and some players may observe (at least partially) the behavior of the others
   - Usually: extensive form for dynamic games and normal form for static games, but it's just convenience, not characteristic of the game; every type of game can get every type of representation

Classification(s) of games (contd.)

3. Perfect, almost perfect, and asymmetric information
   - Dynamic game has **perfect information** if each player, when it's her turn to move, is informed of all previous moves (including the realizations of chance moves)
   - If some moves are simultaneous but each player can observe all past moves, we have **almost perfect information** (or a game with "observable actions")
   - Game with imperfect info has **asymmetric information** if different players have different info on past moves
   - These assumptions are entailed in the rules of the game

Classification(s) of games (contd.)

4. Complete vs incomplete information
   - Event E is **common knowledge** if everybody knows E, everybody knows that everybody knows E, and so on for all iterations of "everybody knows that"
   - Game Γ_N features **complete information** if it's common knowledge that Γ_N is the actual game to be played
   - Conversely, the game features **incomplete information**
   - These are not assumptions on the rules of the game, but on players' interactive knowledge about rules and preferences
   - In most real-world applications, either the outcome function or the players' preferences are not common knowledge

Equilibrium solution concepts

- Rationality not enough to predict what happens
- We must assume beliefs to be mutually consistent
- **Solution concept** = formal rule for predicting the game
- Depending on the game structure we use different equilibrium solution concepts (but be aware that they are just shortcuts of more general hypotheses):

	Complete information	Incomplete information
Static	Nash	Bayesian Nash
Dynamic	Subgame-perfect Nash	Perfect Bayesian

Nash equilibrium

- **Nash equilibrium** $\Rightarrow$ players' beliefs about each other strategies are correct *and* each player best responds to her beliefs. As a result: each player uses strategy that is best response to the strategy used by the others
- Formally:

A strategy profile $s = (s_1, ..., s_I)$ constitutes a Nash equilibrium of the game $\Gamma_N = \langle I, S_i, u_i(.) \rangle$ if for every player $i = 1, ..., I$:

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}), \ \forall s_i' \in S_i$$

Tragedy of the commons (again, prisoner's dilemma)

		US	
		Cooperate	Defect
China	Cooperate	(2,2)	(0,3)
	Defect	(3,0)	(1,1)

- Normal form representation of the game with players, players' strategy spaces, and players' payoff functions
  - Players: US and China / $I = (1,2)$
  - Strategy space: Cooperate, Defect / $S_i = (C,D)$
  - Payoff functions: As indicated by payoff matrix / $u_i = u(s_1, s_2)$

Strategic substitutes (chicken's game)

		France	
		Cooperate	Defect
US	Cooperate	(2,2)	(1,3)
	Defect	(3,1)	(0,0)

- Normal form representation of the game with players, players' strategy spaces, and players' payoff functions
  - Players: US and France / $I = (1, 2)$
  - Strategy space: Cooperate, Defect / $S_i = (C, D)$
  - Payoff functions: As indicated by payoff matrix / $u_i = u(s_1, s_2)$

Strategic complements (assurance dilemma)

		Government	
		Cooperate	Defect
Protesters	Cooperate	(3,3)	(0,2)
	Defect	(2,0)	(1,1)

- Normal form representation of the game with players, players' strategy spaces, and players' payoff functions
  - Players: Protesters and government / $I = (1, 2)$
  - Strategy space: Cooperate, Defect / $S_i = (C, D)$
  - Payoff functions: As indicated by payoff matrix / $u_i = u(s_1, s_2)$

The generals' dilemma (matching pennies)

		Defender	
		Mountains	Plains
Attacker	Mountains	(-1,1)	(1,-1)
	Plains	(1,-1)	(-1,1)

- Normal form representation of the game with players, players' strategy spaces, and players' payoff functions
  - Players: Attacker and defender / $I = (1, 2)$
  - Strategy space: Mountains, Plains / $S_i = (M, P)$
  - Payoff functions: As indicated by payoff matrix / $u_i = u(s_1, s_2)$