Government 2005
Formal Political Theory I
Lecture 3

Instructor: Tommaso Nannicini
Teaching Fellow: Jeremy Bowles

Harvard University

September 14, 2017

Lectures 3: Overview

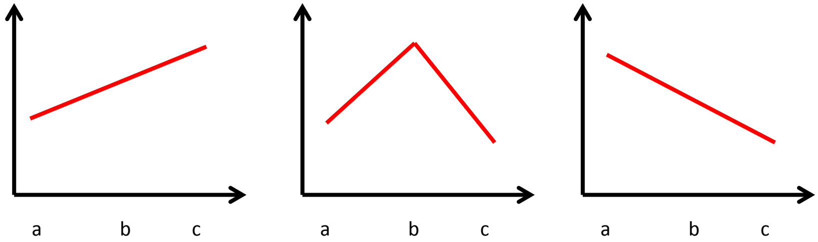
- Median voter theorem: General intuition and discussion
- Hotelling-Downs model of (spatial) electoral competition
- Tragedy of the commons: You'll discuss this with Jeremy

Preliminaries: Condorcet paradox

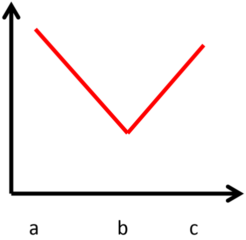
- Example of ordinal preferences of three voters over three alternatives
  - Voter 1: $a \succ_1 b \succ_1 c$
  - Voter 2: $c \succ_2 a \succ_2 b$
  - Voter 3: $b \succ_3 c \succ_3 a$
- There's no Condorcet winner by pairwise majority voting
  - $a \succ_M b \succ_M c \succ_M a$
- Electoral cycles and agenda manipulation
- Even with well-behaved individual preferences (i.e., reflexive, complete, and transitive) we get intransitive aggregate preferences by majority voting
- Possible way out (restricted domain in Arrow's impossibility theorem; just reference for those familiar, we don't care here):
  - Voter 2: $c \succ_2 b \succ_2 a$
  - With these preferences, b is the Condorcet winner

Preliminaries: Single-peaked preferences

- What's the matter with the first version of voter 2's preferences? Answer: They are not single-peaked
- Preferences are single-peaked if: $u(y) > u(z)$ iff $|y - x| < |z - x|$, where x is the bliss point
- Examples of single-peaked preferences:



Example of non-single-peaked preferences



- Applications: Public health care; war in Afghanistan

Preliminaries: Median voter theorem

Median voter theorem. If preferences are single-peaked along a one-dimensional economic policy, the median voter's bliss point represents the equilibrium outcome of the majoritarian voting game

- *Simple proof*. Bliss points of N voters: $(x_1, ..., x_N)$
- N_R voters have $x_i \geq x_m$; N_L voters have $x_i \leq x_m$
- If $N_R \geq N/2$ and $N_L \geq N/2$, x_m is the MV's bliss point
- x_m cannot lose under majority rule. In fact:
- If $z < x_m$ at least $N_R/N \geq 1/2$ vote share in favor of x_m
- If $z > x_m$ at least $N_L/N \geq 1/2$ vote share in favor of x_m

Downs theorem. Suppose a Condorcet winner exists and denote it as x_c. Then, the unique Nash equilibrium of a game in which two candidates/parties compete to win the election is $x_1^* = x_2^* = x_c$

Hotelling-Downs model of electoral competition

- Players: candidates/parties 1 and 2
- Strategies: $x_1, x_2 \in [0, 1]$
- Full commitment to these policy platforms
- Voters sincerely vote for the closest platform
- Voters' bliss points uniformly distributed in $[0, 1]$
- Parties just care about winning (office-seeking)
- Vote share of 1: $(x_1 + x_2)/2$
- Vote share of 2: $1 - (x_1 + x_2)/2$
- Preferences: both parties get W in case of victory, $W/2$ in case of a tie, 0 if they lose

Hotelling-Downs model of electoral competition (contd.)

- $x_1^* = x_2^* = 1/2$ is the unique Nash equilibrium of this game
- To prove this, see that:
  - No incentive to deviate from this equilibrium: otherwise payoff from 0 to -1
  - If there is a tie at $x_1^* = x_2^* \neq 1/2$, both candidates have incentive to deviate to $1/2$: payoff from 0 to 1
  - In all of the other equilibria with no tie, the losing candidate can deviate to $1/2$ and at least tie: payoff from -1 to 0, or from -1 to 1
- In class, we have proved it also by deriving and drawing the best-response correspondence of the two candidates. You should get familiar with this method

Best-response correspondences in Hotelling-Downs

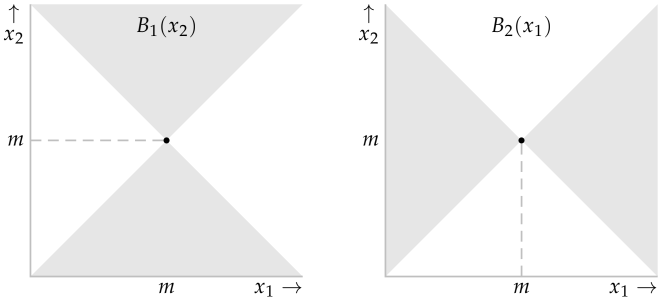


Figure 71.1 The candidates' best response functions in Hotelling's model of electoral competition with two candidates. Candidate 1's best response function is in the left panel; candidate 2's is in the right panel. (The edges of the shaded areas are excluded.)