

# Government 2005: Formal Political Theory I

## Lecture 7

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October 12, 2017

# Overview

- ▶ Sophisticated voting and agenda control
  - ▶ Naive vs sophisticated voting
  - ▶ Power of agenda control
  - ▶ Limits to agenda control
- ▶ Romer-Rosenthal model of agenda control
- ▶ Legislative rules, closed vs open
- ▶ What's next (lecture 8): Repeated games

# Sophisticated voting and agenda control

- ▶ Suppose there are three legislators (1, 2, and 3) and three alternatives ( $A$ ,  $B$ , and  $C$ )
- ▶ The legislators' preferences are as follows:

Legislator 1

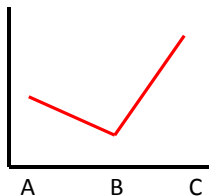
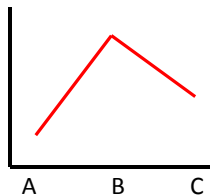
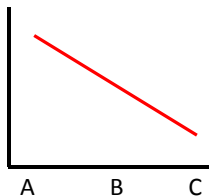
A  
B  
C

Legislator 2

B  
C  
A

Legislator 3

C  
A  
B



## Sophisticated voting and agenda control (contd.)

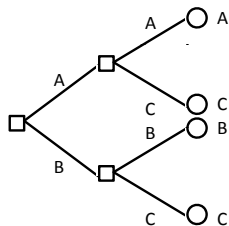
- ▶ Pairwise majority voting would deliver the following outcome/problem
- ▶ Choice between  $A$  and  $B$ , yields  $A$ 
  - ▶ Vote for  $A = \{1, 3\}$ , vote for  $B = \{2\}$
- ▶ Choice between  $A$  and  $C$ , yields  $C$ 
  - ▶ Vote for  $A = \{1\}$ , vote for  $C = \{2, 3\}$
- ▶ Choice between  $C$  and  $B$ , yields  $B$ 
  - ▶ Vote for  $C = \{3\}$ , vote for  $B = \{1, 2\}$
- ▶ No *Condorcet winner*  $\Rightarrow$  Voting cycle

## Sophisticated voting and agenda control (contd.)

- ▶ If there is no *Condorcet winner*, dynamic/sequential voting can be used to achieve an equilibrium in pure strategies (example of **structure-induced equilibrium**)
- ▶ However, we trade one problem for another: Whoever controls the agenda (voting timing) will have proposal power and will be able to influence the outcome (with or without constraints depending on the players' preferences)

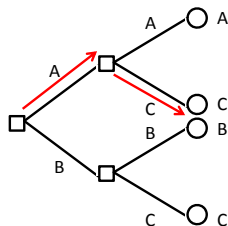
## Sophisticated voting and agenda control (contd.)

- ▶ Suppose the legislators must choose either  $A$ ,  $B$ , or  $C$ , by majority voting with a fixed agenda
- ▶ The agenda establishes that they first choose between  $A$  and  $B$ , and then the winner is paired against  $C$
- ▶ Then the “voting tree” is the following:



## Sophisticated voting and agenda control (contd.)

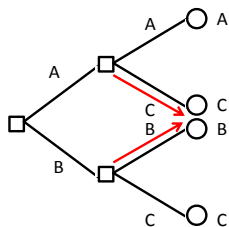
- ▶ Assume the legislators vote naively, without looking ahead down the voting tree
- ▶ In the first round,  $A$  beats  $B$ 
  - ▶ Vote for  $A = \{1, 3\}$ , vote for  $B = \{2\}$
- ▶ In the last round,  $C$  beats  $A$ 
  - ▶ Vote for  $A = \{1\}$ , vote for  $C = \{2, 3\}$



- ▶  $C$  is the winning policy

## Sophisticated voting and agenda control (contd.)

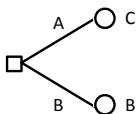
- ▶ Suppose the legislators vote sophisticatedly, looking ahead down the voting tree
- ▶ We then look for a backward-induction outcome
- ▶ In the last round, the legislators all vote sincerely
- ▶ Choice between  $A$  and  $C$ , yields  $C$ 
  - ▶ Vote for  $A = \{1\}$ , vote for  $C = \{2, 3\}$
- ▶ Choice between  $B$  and  $C$ , yields  $B$ 
  - ▶ Vote for  $B = \{1, 2\}$ , vote for  $C = \{3\}$



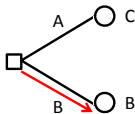


## Sophisticated voting and agenda control (contd.)

- ▶ This means the vote in the first round is *really* a vote between  $C$  and  $B$ :



- ▶ We just saw that a majority of the legislators prefer  $B$  to  $C$ , so  $B$  will win in the first round, and will be the final choice

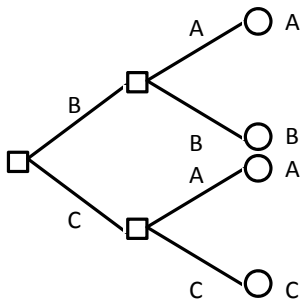


## Sophisticated voting and agenda control (contd.)

- ▶ What's going on?
- ▶ In the first round, Legislator 1 casts a “sophisticated” vote for  $B$ , in order to insure that  $B$  is the policy choice
- ▶ If Legislator 1 were to vote “sincerely” in the first round, and vote for  $A$ , then  $C$  would be the policy choice, and  $C$  is her *least* favorite policy
- ▶ But Legislator 1 votes for  $B$  instead of  $C$ , and at least gets her second choice
- ▶ Note that for Legislators 2 and 3, there is no difference between voting strategically and sincerely
- ▶ Empirical work scholars often find that very little “sophisticated voting” appears to occur in practice, but it does not mean that legislators are acting naively

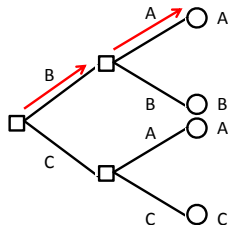
## Sophisticated voting and agenda control (contd.)

- ▶ We now assume the agenda is different
- ▶ The legislators first choose between  $B$  and  $C$ , and then the winner is paired against  $A$
- ▶ The “voting tree” is the following:



## Sophisticated voting and agenda control (contd.)

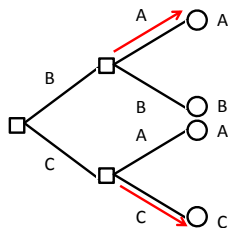
- ▶ Suppose again that the legislators vote naively, without looking ahead down the voting tree
- ▶ In the first round,  $B$  beats  $C$ 
  - ▶ Vote for  $B = \{1, 2\}$ , vote for  $C = \{3\}$
- ▶ In the last round,  $A$  beats  $B$ 
  - ▶ Vote for  $A = \{1, 3\}$ , vote for  $B = \{2\}$



- ▶  $A$  is the winning policy

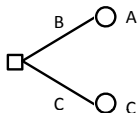
## Sophisticated voting and agenda control (contd.)

- ▶ Suppose again that legislators vote sophisticatedly, looking ahead down the voting tree
- ▶ We then look for a backward-induction outcome
- ▶ In the last round, the legislators all vote sincerely
- ▶ Choice between  $A$  and  $C$ , yields  $C$ 
  - ▶ Vote for  $A = \{1\}$ , vote for  $C = \{2, 3\}$
- ▶ Choice between  $A$  and  $B$ , yields  $A$ 
  - ▶ Vote for  $A = \{1, 3\}$ , vote for  $B = \{2\}$

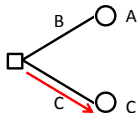


## Sophisticated voting and agenda control (contd.)

- ▶ This means the vote in the first round is *really* a vote between A and C:



- ▶ We just saw that a majority of the legislators prefer C to A, so C will win in the first round, and will be the final choice



## Sophisticated voting and agenda control (contd.)

- ▶ In the first round, Legislator 2 casts a “sophisticated” vote for  $C$ , in order to insure that  $C$  is the policy choice
- ▶ If Legislator 2 were to vote “sincerely” in the first round, and vote for  $B$ , then  $A$  would be the policy choice, and  $A$  is her *least* favorite policy
- ▶ Legislator 2 votes for  $C$  instead, and at least gets her second choice
- ▶ **Thus, the agenda has a crucial impact on the ultimate policy outcome**

## Sophisticated voting and agenda control (contd.)

- ▶ The control of the agenda is potentially an important source of political power
- ▶ But what are the limits to this power (if any)?
- ▶ The answer is given by the “uncovered set”
- ▶ Alternative  $B$  is *covered* by  $A$  if  $A$  defeats  $B$  in a pairwise majority vote (under sincere voting), and if  $A$  also defeats any point that  $B$  defeats
- ▶ If  $A$  covers  $B$ , we cannot construct an agenda in which  $B$  wins
- ▶ An alternative is *uncovered* if no alternatives cover it

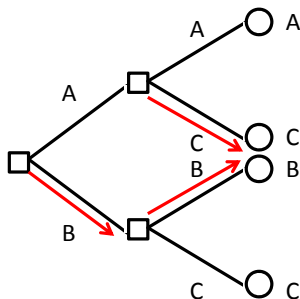


## Sophisticated voting and agenda control (contd.)

- ▶ The intuition is simple
- ▶ Consider the case where  $A$  defeats  $B$  in a pairwise majority vote (under sincere voting)
- ▶ Say we want to construct an agenda in which  $B$  wins
- ▶ The strategy is to put  $A$  down the tree so that is beaten by some other policy  $C$  that later  $B$  can beat up the tree
- ▶ However, when  $A$  covers  $B$ ,  $A$  also defeats any point that  $B$  defeats, and it's thus impossible to find such a policy  $C$

## Sophisticated voting and agenda control (contd.)

- ▶ This is exactly the trick used by the agenda setter in the first example (as  $B$  was not covered by  $A$ )



## Sophisticated voting and agenda control (contd.)

- ▶ A particular example where  $A$  covers  $B$ :

Leg. 1	Leg. 2	Leg. 3	Leg. 4	Leg. 5
A	A	D	D	C
B	C	B	A	B
C	D	A	B	D
D	B	C	C	A

- ▶  $A$  defeats  $B$  and  $C$ ,  $B$  defeats  $C$ ,  $C$  defeats  $D$  and  $D$  defeats  $A$  and  $B$
- ▶ There is a majority-cycle containing all points, but  $A$  covers  $B$

## Sophisticated voting and agenda control (contd.)

- ▶ Since  $A$  covers  $B$ , we cannot construct an agenda in which  $B$  wins
- ▶ To see this, suppose  $B$  wins
- ▶ At each level in the voting tree,  $B$  must defeat the alternative at that level
- ▶ But  $A$  will also defeat the alternative at that level (since, by the definition of covering,  $A$  defeats everything that  $B$  defeats)
- ▶ Finally, at the top,  $B$  will confront  $A$
- ▶ But at that point  $A$  will win, because  $A$  defeats  $B$  because a majority prefers it
- ▶ So,  $B$  cannot win: If  $B$  is the bliss point of the agenda setter, he cannot get it (because it's not in the uncovered set)

## Sophisticated voting and agenda control (contd.)

- ▶ We can instead construct agendas in which  $A$  wins
- ▶ To do this, we must make sure that  $D$  (the only alternative that defeats  $A$ ) is defeated (by  $C$ ) before  $A$  confronts it
- ▶ The voting order ( $B, D, A, C$ ) would work
  - ▶  $C$  would defeat  $D$  in the last round, but  $A$  would defeat  $C$  in that round, and thus the majority prefers voting  $A$  to  $D$
  - ▶  $A$  would also beat  $B$  in the first round
- ▶ If  $A$  is the agenda setter's bliss point, the agenda setter is unconstrained and can get what he wants

# Romer-Rosenthal model

- ▶ Assume a proposer (e.g., a school board) must propose a spending level  $x \in [0, \infty)$
- ▶ The proposal is then voted against the status quo  $s$  in a referendum
- ▶ Proposer's utility:  $u_p(x)$  with  $u_p(\cdot)$  strictly increasing
- ▶ There are  $N$  voters (with  $N$  odd), who vote by majority rule
  - ▶ Their strategies are:  $\{Y, N\}$  (full turnout)
  - ▶ They have single-peaked preferences with bliss point  $v_i$ :  
 $u_i(x) = h(-|x - v_i|)$  with  $h(\cdot)$  strictly increasing
- ▶ If the vote share of  $Y$  is larger than  $N$ ,  $x$  is implemented; otherwise, the status quo  $s$  is implemented

## Romer-Rosenthal model (contd.)

- ▶ The last stage is a majority-rule voting game
- ▶ Voting for the preferred alternative is not strictly dominated
  - ▶ Unless  $(N - 1)/2$  votes for  $Y$  and  $(N - 1)/2$  votes for  $N$ ,  $i$ 's utility doesn't depend on her vote
- ▶ But voting for the preferred alternative weakly dominates voting for the less preferred
- ▶ Under weakly undominated strategies, each voter votes for her preferred alternative, that is:
  - ▶  $Y$  if  $u_i(x) \geq u_i(s)$ , or  $N$  otherwise
- ▶ Therefore, under weakly undominated strategies,  $x$  beats  $s$  if the median voter prefers  $x$  to  $s$ , and  $s$  beats  $x$  otherwise

## Romer-Rosenthal model (contd.)

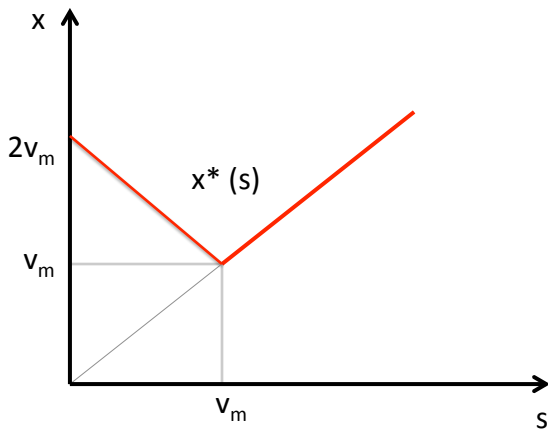
- ▶ Let  $v_m$  be the bliss point of the median voter
- ▶ The policies voted by the median voter must satisfy  $u_m(x) \geq u_m(s)$ , that is:
  - ▶  $|x - v_m| \leq |s - v_m|$
- ▶ This implies that:
  - ▶ If  $s < v_m \rightarrow x \in [s, 2v_m - s]$
  - ▶ If  $s > v_m \rightarrow x \in [2v_m - s, s]$
- ▶ As a result, going backward to the first stage of the game, the proposer will set her optimal policy such that:
  - ▶  $x^*(s) = \max\{s, 2v_m - s\}$



# Romer-Rosenthal model (contd.)

## Equilibrium policies

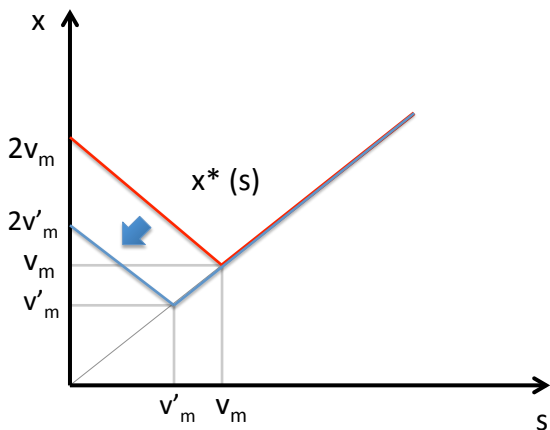
- ▶ The equilibrium proposal as a function of the status quo is:



# Romer-Rosenthal model (contd.)

## Comparative statics

- ▶ The median voter's bliss point has a clear impact on the equilibrium proposals:



# Legislative rules

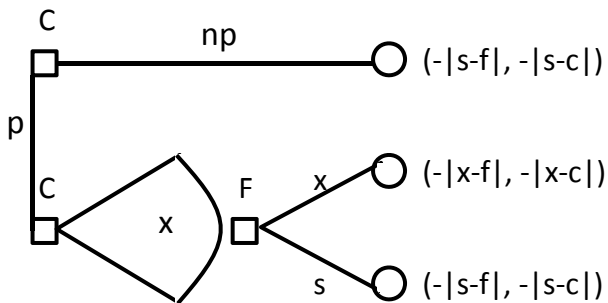
## Closed rule

- ▶ Consider the following game of legislative decision making
- ▶ The players are  $C$ , a committee, and  $F$ , the floor/assembly
- ▶ There is a one-dimensional policy space, and the players have symmetric single-peaked preferences:
  - ▶  $u_C(x) = -|x - c|$  for the committee
  - ▶  $u_F(x) = -|x - f|$  for the floor (e.g., median legislator)
- ▶ There is a status quo policy,  $s$
- ▶ The committee may propose a bill,  $x$ , or it may do nothing
- ▶ If the committee proposes a bill, and the bill is considered under a **closed rule**, then the median legislator simply chooses between  $x$  and  $s$

# Legislative rules (contd.)

## Closed rule

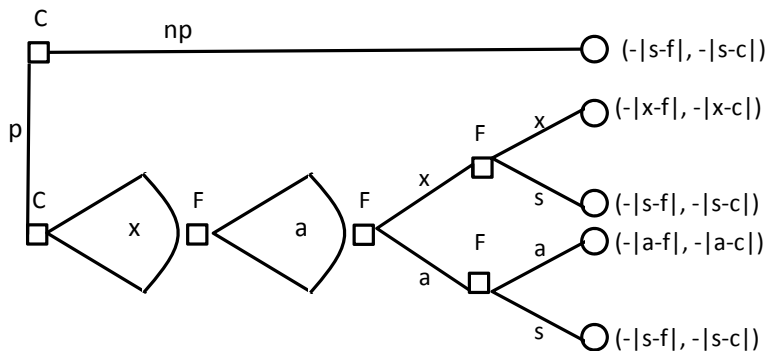
- ▶ The game tree is the following:



# Legislative rules (contd.)

## Open rule

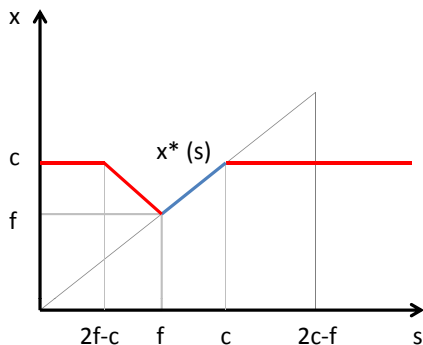
- ▶ Instead, if after the committee's proposal, the bill is considered under an **open rule**, then the floor can amend the bill to  $a$ , and choose among  $a$  vs  $x$ , and then  $s$ :



# Legislative rules (contd.)

## Closed rule

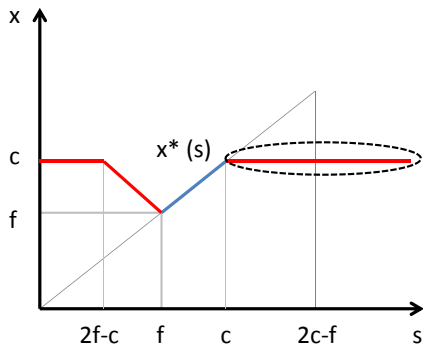
- ▶ Assume  $c > f$
- ▶ Under the closed rule, the committee's optimal bill choice  $x^*(s)$  as a function of the status quo is as follows:



# Legislative rules (contd.)

## Closed rule

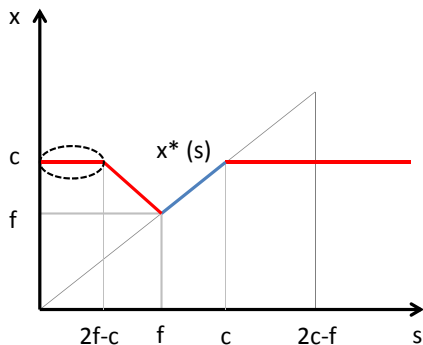
- ▶ If  $s > c$  the committee proposes its ideal point  $c$  and the floor accepts it



# Legislative rules (contd.)

## Closed rule

- ▶ If  $s < 2f - c$  the committee proposes its ideal point  $c$  and the floor accepts it, since  $-|c - f| > -|s - f|$  as long as  $s < 2f - c$

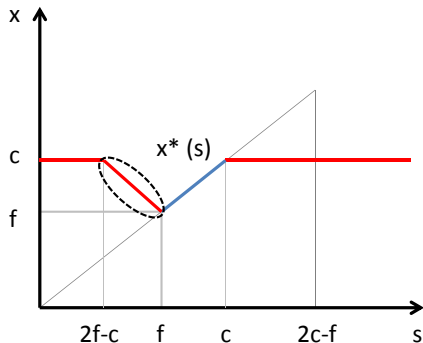




# Legislative rules (contd.)

## Closed rule

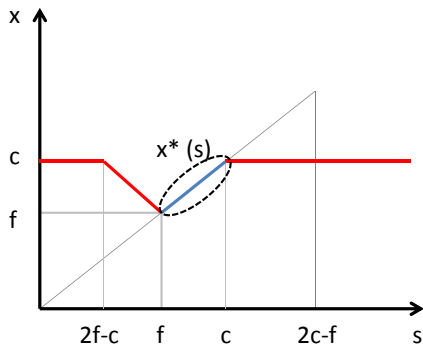
- ▶ If  $s \in (2f - c, f)$  the committee proposes  $x$  as close as possible to  $c$  such that  $-|x - f| = -|s - f|$



# Legislative rules (contd.)

## Closed rule

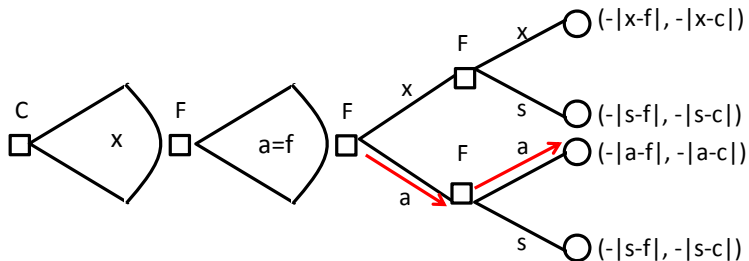
- ▶ If  $s \in (f, c)$ , there is no  $x$  that the committee would like to propose that the floor would accept



# Legislative rules (contd.)

## Open rule

- ▶ Under open rule,  $(f, a, a)$  is a dominant strategy for the floor



# Legislative rules (contd.)

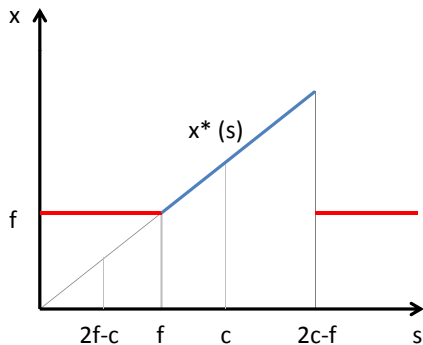
## Open rule

- ▶ The committee can report *any* bill, because it knows the floor
  - ▶ will offer the amendment  $a = f$
  - ▶ will choose  $a$  to  $x$
  - ▶ and will choose  $a$  to  $s$
- ▶ As a result, the final outcome will be  $f$
- ▶ So, it might as well just report  $f$  to begin with, whenever it reports a bill

# Legislative rules (contd.)

## Open rule

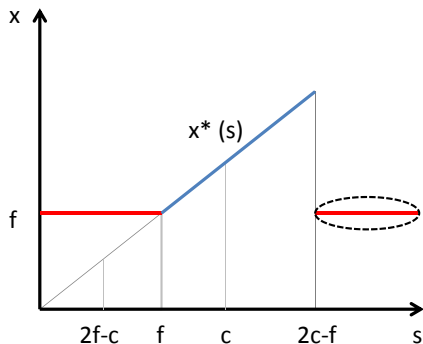
- ▶ Assume  $c > f$
- ▶ Under the open rule, the committee's optimal bill choice is as follows:



# Legislative rules (contd.)

## Open rule

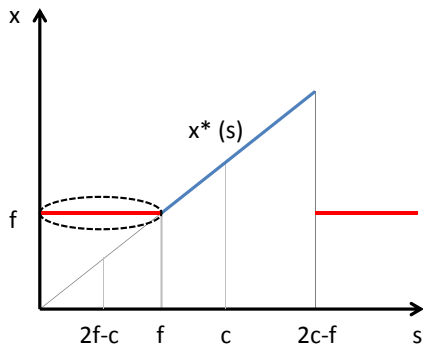
- ▶ If  $s > 2c - f$  the committee proposes  $f$  and the floor accepts
- ▶ The committee has the incentive to do so since  $-|f - c| > -|s - c|$  as long as  $s > 2c - f$



# Legislative rules (contd.)

## Open rule

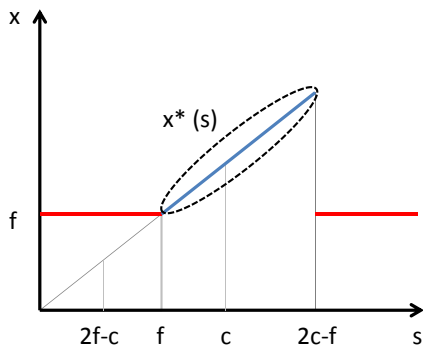
- ▶ Also the committee strictly prefers  $f$  to  $s < f$



# Legislative rules (contd.)

## Open rule

- ▶ If  $s \in (f, 2c - f)$  the committee has no chance to make an acceptable proposal as the floor would go with  $a = f$
- ▶ Hence, it makes no proposal and stays with the status quo





# Legislative rules (contd.)

## Final remarks

- ▶ Under open rule, the final outcome is  $f$  whenever the committee reports a bill, while under closed rule the final outcome is more likely to be  $c$  than anything else
- ▶ Under open rule the committee is more likely to report *no* bill at all
  - ▶ Under both rules, the committee does not report a bill when  $s \in [f, c]$
  - ▶ Under the open rule, the committee also does not report a bill when  $s \in (c, 2c - f)$